**Intermediate test Waves and Optics - 8 December 2014**

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**This test contains 4 questions on 3 pages.**

A few preliminary remarks:

- Please answer questions 3 & 4 on another (double) sheet of paper than questions 1 & 2.
- Put your name and student number at the top of all sheets.
- Put your student card at the edge of the desk for checking by the assistants and show it when handing in your test.

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**Question 1 (5 points): Spherical waves**

In the case of spherical waves in an isotropic medium, the wave function $\psi(r,t)$ only depends on the spherical coordinate $r$ (the distance to the center of the spherical wavefronts) and time $t$. In this case, the 3-dimensional differential wave equation can be written as:

$$\frac{1}{r}\frac{\partial^2(r\psi)}{\partial r^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

with v the speed of the wave.

*Questions:*

a) Which property of the amplitude of a spherical wave follows from the equation above (and explain how this follows from the equation) ?

b) Write down an expression for:
   b.1 the wave function of a spherical harmonic wave that originates from a point ("exploding");
   b.2 the wave function of a spherical harmonic wave that moves towards a point ("imploding").

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(Question 2 on the next page)

**Question 2 (5 points): Superposition of waves**

The superposition of $N$ waves with the same frequency ( $\omega$ ) but with different amplitudes ( $E_{0i}$ ) and initial phases ( $\alpha_i$ ), $E = \sum_{i=1}^{N} E_{0i} \cos(\alpha_i \pm \omega t)$, can be written as:

$$E = E_0 \cos(\alpha \pm \omega t)$$

with:

$$E_0^2 = \sum_{i=1}^{N} E_{0i}^2 + 2 \sum_{j>i}^{N} \sum_{i=1}^{N} E_{0i} E_{0j} \cos(\alpha_i - \alpha_j)$$

$$\tan \alpha = \frac{\sum_{i=1}^{N} E_{0i} \sin \alpha_i}{\sum_{i=1}^{N} E_{0i} \cos \alpha_i}$$

The irradiance $I$ of a harmonic wave is given by: $I = \frac{v \varepsilon}{2} E_0^2$, with v the velocity and $E_0$ the amplitute of the wave and $\varepsilon$ the permittivity of the medium in which the wave propagates.

*Questions:*

Assume that all waves have the same amplitude ( $E_{0i} = E_{01}$ ) and are coherent.

a) Derive an expression for the irradiance of the superposition as a function of $E_{01}$ and $N$ in a point where all waves are in phase. Include in the answer what is meant by the waves being coherent.

b) Discuss the result in the context of the principle of conservation of energy.

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(Question 3 on the next page)

**Question 3 (5 points): Harmonic waves**

Consider a plane harmonic light wave with an electric field given by $E_z = E_0 \sin\left[\pi 10^{15}\left(t - \frac{x}{0.75c}\right)\right]$ while travelling in a certain transparent medium ($c$ represents the speed of light in vacuum).

*Questions:*

a) What is the frequency of the light ?
b) What is the wavelength of the light ?
c) What is the index of refraction of the medium for this wave ?

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**Question 4 (5 points): Total internal reflection**

Consider a beam of light incident on an interface in a situation of internal reflection. When the angle of incidence is larger than the so-called critical angle, total internal reflection occurs.

*Question:*

What is the value of the critical angle for an interface between water (index of refraction $n = 1.33$) and flint glass ($n = 1.84$) ?

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